Now for as much as on the same Focus, may be drawn an infinite Number of Parabola's; and to every of those, an infinite number of Hyperbola's, whose Center shall be that Focus; it follows that there is a variety doubly infinite of such pairs of Equal Isoperimetral Equicrural Triangles. And all this, without varying the Point F, or the Plain FSV, or the Position of FA in that Plain; every of which will yet afford an infinite variety. And yet even this may be infinitely varied, if we leave out the condition of Equicrural. For then a new Semi-Parabole with a new Hyperbola, will give new portions (on the other side of the Perpendicular FR,) of such Equal Isoperimeter Triangles, (but not Equicrural) with infinite variety.

Of this nature is that Problem which *Francis van Schooten* tells us, (in the Twelfth Section of his *Sectiones Miscellaneæ*,) was openly proposed at *Paris*, in the Year 1633: *To find Two Equicrural Triangles, equal each to other in Perimeter and Area*, but further clogged with this condition; *so that all their Sides and Perpendiculars be commensurable*, or as Number to Number. Which new condition doth restrain the Problem, but not determine it; so that it is yet capable of innumerable Solutions.

To this, he tells us, *Des Cartes* gave one Solution, (making the Sides of the one 29, 29, 40; of the other 37, 37, 24;) But Dr. *John Pell*, (in his Introduction to Algebra, published by *Thomas Brancker*, at his *Probl.* 29. 30, 31.) discusseth the same at large; and shews how, by easy Methods, (from Tables by him set down,) to give innumerable Solutions in Integer Numbers.

And he shews moreover (which is very true,) that to every of these pairs, there belongs a Third Triangle; whose Base if supposed a Negative quantity, the Aggregate of it, and the Legs, will be equal to the Sum of the Base and Legs in either of the other: Or (which is all one) the Legs wanting the Base (if in this the Base be supposed Affirmative:) will be Equal to the Sum of the Base and Legs in either of the other. As for instance, (in *Des Cartes*'s Triangle,) $= 29, + 29, + 40, = 37, + 37, + 24, (= 98) = 56\frac{1}{2}, + 56\frac{1}{2}, - 15$; the Perimeter. And the Perpendiculars will be $21, 35, - 56$. And consequently, $21 \times 40 = 35 \times 24 = - 56 \times - 15$ $(= 840)$ the Double of the Area. The whole Process I forbear to repeat; referring to the Author for it.

CHAP. LXVI.

Of NEGATIVE SQUARES, *and their* IMAGINARY ROOTS *in Algebra.*

We have before had occasion (in the Solution of some Quadratick and Cubick Equations) to make mention of Negative Squares, and Imaginary Roots, (as contradistinguished to what they call Real Roots, whether Affirmative or Negative:) But referred the fuller consideration of them to this place.

These *Imaginary* Quantities (as they are commonly called) arising from the *Supposed* Root of a Negative Square, (when they happen,) are reputed to imply that the Case proposed is Impossible.

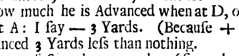
And so indeed it is, as to the first and strict notion of what is proposed. For it is not possible, that any Number (Negative or Affirmative) Multiplied into itself, can produce (for instance) -4. Since that Like Signs (whether $+$ or $-$) will produce $+$; and therefore not -4.

But it is also Impossible, that any Quantity (though not a Supposed Square) can be *Negative*. Since that it is not possible that any *Magnitude* can be *Less than Nothing*, or any *Number Fewer than None*.

Yet

Yet is not that Supposition (of Negative Quantities,) either Unuseful or Absurd; when rightly understood. And though, as to the bare Algebraick Notation, it import a Quantity less than nothing: Yet, when it comes to a Physical Application, it denotes as Real a Quantity as if the Sign were $+$; but to be interpreted in a contrary sense.

As for instance: Supposing a man to have advanced or moved forward, (from A to B,) 5 Yards; and then to retreat (from B to C) 2 Yards: If it be asked, how much he had Advanced (upon the whole march) when at C? or how many Yards he is now Forwarder than when he was at A? I find (by Subducting 2 from 5,) that he is Advanced 3 Yards. (Because $+5 - 2 = +3$.)



But if, having Advanced 5 Yards to B, he thence Retreat 8 Yards to D; and it be then asked, How much he is Advanced when at D, or how much Forwarder than when he was at A: I say -3 Yards. (Because $+5 - 8 = -3$.) That is to say, he is advanced 3 Yards less than nothing.

Which in propriety of Speech, cannot be, (since there cannot be less than nothing.) And therefore as to the Line AB *Forward*, the case is Impossible.

But if (contrary to the Supposition,) the Line from A, be continued *Backward*, we shall find D, 3 Yards *Behind* A. (Which was presumed to be *Before* it.)

And thus to say, he is *Advanced* -3 Yards; is but what we should say (in ordinary form of Speech,) he is *Retreated* 3 Yards; or he wants 3 Yards of being so Forward as he was at A.

Which doth not only answer Negatively to the Question asked. That he is not (as was supposed,) Advanced at all: But tells moreover, he is so far from being Advanced, (as was supposed,) that he is Retreated 3 Yards; or that he is at D, more Backward by 3 Yards, than he was at A.

And consequently -3, doth as truly design the Point D; as $+3$ designed the Point C. Not Forward, as was supposed; but Backward, from A.

So that $+3$, signifies 3 Yards Forward; and -3, signifies 3 Yards Backward: But still in the same Streight Line. And each designs (at least in the same Infinite Line,) one Single Point: And but one. And thus it is in all Lateral Equations; as having but one Single Root.

Now what is admitted in Lines, must on the same Reason, be allowed in Plains also.

As for instance: Supposing that in one Place, we Gain from the Sea, 30 Acres, but Lose in another Place, 20 Acres: If it be now asked, How many Acres we have gained upon the whole: The Answer is, 10 Acres, or $+10$. (Because of $30 - 20 = 10$.) Or, which is all one 1600 Square Perches. (For the *English* Acre being Equal to a Plain of 40 Perches in length, and 4 in breadth, whose Area is 160; 10 Acres will be 1600 Square Perches.) Which if it lye in a Square Form, the Side of that Square will be 40 Perches in length; or (admitting of a Negative Root,) -40.

But if then in a Third place, we lose 20 Acres more; and the same Question be again asked, How much we have gained in the whole; the Answer must be -10 Acres. (Because $30 - 20 - 20 = -10$.) That is to say, The Gain is 10 Acres less than nothing. Which is the same as to say, there is a Loss of 10 Acres: or of 1600 Square Perches.

And hitherto, there is no new Difficulty arising, nor any other Impossibility than what we met with before, (in supposing a Negative Quantity, or somewhat Less than nothing:) Save only that $\sqrt{1600}$ is ambiguous; and may be $+40$, or -40. And from such Ambiguity it is, that Quadratick Equations admit of Two Roots.

But now (supposing this Negative Plain, -1600 Perches, to be in the form of a Square;) must not this Supposed Square be supposed to have a Side? And if so, What shall this Side be?

We